Simultaneous Heat and Mass Transfer in Free Convection from a Horizontal Cylinder

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Steady laminar boundary-layer analysis of combined heat and mass transfer characteristics in free convection around a horizontal cylinder has been carried out by two approximate methods, viz., local similarity and local nonsimilarity approaches. Numerical results of the local Nusselt number, the local Sherwood number, and local surface shear stress are presented, along with the dimensionless thermal, mass fraction, and momentum boundary-layer thicknesses in tabular form. The parameter ranges are chosen to encompass most commonly occurring circumstances in natural and technological processes.

Nomenclature

C = species mass fraction or concentration, dimensionless
D = diameter of the cylinder, m
D^* = binary diffusion coefficient, m²/s
f = reduced stream function, dimensionless
g = acceleration due to gravity, m/s²
Gr_D = local thermal Grashof number ($=g\beta(T_w-T_\infty)D^3/\nu^2$), dimensionless
$Gr_{D,c}$ = local Grashof number due to mass diffusion ($=g\beta^*(C_w-C_\infty)D^3/\nu^2$)
k = thermal conductivity of fluid, W/mK
$\dot{m}$ = mass flow rate, kg/m²s
$\dot{m}_w$ = mass flux of diffusing species, kg/m²
N = ratio of concentration to thermal Grashof numbers ($Gr_{D,c}/Gr_D$), dimensionless
q = dimensionless temperature $(T-T_\infty)/(T_w-T_\infty)$
q_w = local wall heat flux, W/m²
Sc = Schmidt number ($=\nu/D^*$)
Sh_D = local Sherwood number ($=m_w D/\rho D^*(C_w-C_\infty)$)
T = temperature of the fluid, K
u,v = velocity components in x and y directions, m/s
U,V = dimensionless velocity components
x,y = local streamwise and normal coordinates, m
X,Y = dimensionless streamwise and normal coordinates
α = thermal diffusivity, m²/s
β = volumetric coefficient of thermal expansion, $-(\partial\rho/\partial T)_{p,c}/\rho$, K
β^* = volumetric coefficient of expansion with mass fraction $-(\partial\rho/\partial C)_{p,T}/\rho$
δ_c = dimensionless concentration boundary layer
δ_t = dimensionless thermal boundary layer
δ_v = dimensionless velocity boundary layer
θ = angular distance from the stagnation point, rad
η = pseudo-similarity variable, dimensionless
η_∞ = final limit of integration, dimensionless
μ = dynamic viscosity, NS/m²
ν = kinematic viscosity of fluid, m²/s
ω = dimensionless concentration ($=(C-C_\infty)/(C_w-C_\infty)$)
ρ = density of fluid, kg/m³
ξ = transformed streamwise coordinate, dimensionless
Λ = configuration function defined in Eq. (13), dimensionless
τ = shear stress, N/m²
ψ = stream function, dimensionless

Superscripts

$(\ )'$ = differentiation with respect to η

Subscripts

w = condition at the wall
∞ = condition at the edge of the thickest boundary layers among concentration, thermal and velocity

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Introduction

FREE convection with simultaneous transport of energy and species plays an important role in a number of practical and technological processes, e.g., drying of grains, control of surface temperature by evaporative cooling and transpiration, melting of ice in saline waters, and curing of the surface of materials. Heat and mass transfer by natural convection from moist air to horizontal tubes is encountered in many practical applications in refrigeration and air conditioning.

Despite an outpouring of literature dealing with a single buoyancy force mechanism, there is a dearth of literature dealing with flows driven by multiple buoyancy forces. This has already been identified.[1] It is well known that for simultaneous natural convection heat and mass transfer, the transfer coefficients are not related to those for either pure heat transfer or pure mass transfer by the oft-used heat/mass transfer analogy. The natural convection literature for pure heat transfer or pure mass transfer about cylinders is therefore not useful in the case of simultaneous transfer.

Previous studies considering multiple buoyancy forces include vertical,[2,3] horizontal,[4,5] and inclined[6] surfaces. A detailed literature search failed to reveal any analytic or experimental study of combined diffusion of heat and chemical species in natural convection around a horizontal cylinder. The purpose of the present study is to obtain a systematic synopsis of the mass, heat, and momentum transfer characteristics around a horizontal circular cylinder under free convection.

Analysis

Consideration is given here to the steady, laminar, free-convection boundary-layer flow around a long, horizontal circular cylinder of diameter D. The cylinder is situated in an infinite ambient fluid of undisturbed temperature and concentration, T_∞ and C_∞, respectively. The surface of the cylinder is maintained at uniform temperature T_w and uniform concentration C_w. Let the local orthogonal coordinates be chosen such that x measures the distance along the surface of the cylinder from the lower stagnation point and y the distance normal to the surface into the fluid. Under the assumption of incompressible flow, negligible dissipation, and constant properties (except for density changes which produce buoyancy forces), the governing boundary-layer equations after neglecting the curvature effects of the cylinder are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\left(\frac{\partial u}{\partial x}\right)+v\left(\frac{\partial u}{\partial y}\right)=\nu\left(\frac{\partial^2 u}{\partial y^2}\right)+g\beta(T-T_\infty)\sin\left(\frac{2x}{D}\right)+g\beta^*(C-C_\infty)\sin\left(\frac{2x}{D}\right) \tag{2}$$

$$u\left(\frac{\partial T}{\partial y}\right)+v\left(\frac{\partial T}{\partial y}\right)=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

$$u\left(\frac{\partial C}{\partial y}\right)+v\left(\frac{\partial C}{\partial y}\right)=D^*\left(\frac{\partial^2 C}{\partial y^2}\right) \tag{4}$$

The symbols are given in the Nomenclature. The second and third terms on the right-hand side of Eq. (2) represent the buoyancy forces arising, respectively, from the temperature and concentration variations in the fluid. The boundary conditions for Eqs. (1-4) are

$$\begin{aligned}&u=0,\quad v=v_w,\quad T=T_w,\quad C=C_w \text{ at } y=0\\&u\to 0,\quad T\to T_\infty,\quad C\to C_\infty \text{ at } y\to\infty\\&u=0,\quad T=T_\infty,\quad C=C_\infty \text{ at } X=0 \text{ and } y>0\end{aligned} \tag{5}$$

The conservation equations can be recast into their dimensionless form by introducing the diameter of the sphere D as the reference length, appropriate stretched coordinates, and normalizing velocities, temperature, and concentration as defined below:

$$q=\frac{T-T_\infty}{T_w-T_\infty},\quad \omega=\frac{C-C_\infty}{C_w-C_\infty},\quad X=\frac{x}{D},\quad Y=\frac{yGr_D^{1/4}}{D}$$
$$U=uD/[\nu Gr_D^{1/2}],\qquad V=vD/[\nu Gr_D^{1/2}] \tag{6}$$

where the Grashof number $Gr_D=g\beta(T_w-T_\infty)D^3/\nu^2$. To facilitate the analysis, we introduce the stream function ψ, such that

$$U=\frac{\partial\psi}{\partial Y} \text{ and } V=-\frac{\partial\psi}{\partial X} \tag{7}$$

and consider the following transformations[7]

$$\xi=\sqrt{2}(1-\cos X) \tag{8a}$$

$$\eta=Y\sin X/(\sqrt{\xi}) \tag{8b}$$

$$\psi=\sqrt{2\xi}f(\xi,\eta) \tag{8c}$$

Using the above transformations, Eqs. (1-4) become

$$f'''+ff''+\Lambda(\xi)[-(f')^2+q+N\omega]=2\xi\frac{\partial(f',f)}{\partial(\xi,\eta)} \tag{9}$$

$$q''+fq'Pr=2\xi\frac{\partial(q,f)}{\partial(\xi,\eta)}Pr \tag{10}$$

$$\omega''+f\omega' Sc=2\xi\frac{\partial(\omega,f)}{\partial(\xi,\eta)}Sc \tag{11}$$

and the boundary conditions become

$$f'(\xi,0)=q(\xi,0)-1=\omega(\xi,0)-1=0 \tag{12a}$$

$$f(\xi,0)+2\xi\left(\frac{\partial f}{\partial\xi}\right)=0 \tag{12b}$$

$$f'(\xi,\infty)=q(\xi,\infty)=\omega(\xi,\infty)=0 \tag{12c}$$

where the function $\Lambda(\xi)$ is defined by

$$\Lambda(\xi)=(2\sqrt{2}-2\xi)/(2\sqrt{2}-\xi) \tag{13}$$

and

$$N=Gr_{D,c}/Gr_D=\beta^*(C_w-C_\infty)/[\beta(T_w-T_\infty)] \tag{14}$$

The boundary condition, Eq. (12b), is written with the assumption that the species concentration is low and hence the normal component of interfacial velocity is neglected. The condition for the neglect of the normal component of the interfacial velocity can be shown to be

$$2^{3/4}v_w D\sqrt{\xi}/[\nu Gr_D^{1/4}(1-\cos\theta)^{1/2}]\ll 1 \tag{15}$$

Using the condition of no mass transfer of the inert component at the interface and applying Fick's law, it can be shown that the transverse velocity at the wall is given by

$$v=v_w=-\frac{D^*(\partial C/\partial y)_w}{(1-C_w)} \tag{16}$$

Using Eqs. (15) and (16), the following alternative condition can be derived for the neglect of the contribution of the transverse velocity at the surface to the interface mass transfer

$$(C_w-C_\infty)[-\omega'(\xi,0)]/[Sc(1-C_w)]\ll 1 \tag{17}$$

Often in natural convection processes with low mass fraction the condition shown in Eq. (17) is fully satisfied and the neglect of the interfacial velocity is justified.[5]

The problem defined by Eqs. (9-14) could be solved by the finite difference scheme, for example, by a Keller Box type of scheme.[8] In the present analysis the above equations were solved by local similarity (LS) and two-equation local nonsimilarity approaches. The latter two methods were used very successfully in solving a number of free-convective boundary-layer problems and are also well documented in the literature.[9,10]

In the local similarity approach the derivatives of ξ terms in Eqs. (9-12) are neglected. This approximation is justified for small values of ξ or the derivatives of the quantities f, f', q, and ω with respect to ξ. Nevertheless, it will be seen that the results obtained when the above condition is not satisfied also give results sufficiently accurate for estimation purposes. Thus the truncated equations become

$$f'''+ff''+\Lambda(\xi)[-(f')^2+q+N\omega]=0 \tag{18}$$

$$q''+fq'Pr=0 \tag{19}$$

$$\omega''+f'_\omega Sc=0 \tag{20}$$

All the boundary conditions remain unchanged except Eq. (12b), which simplifies to $f(\xi,0)=0$. Equations (18-20) are ordinary coupled nonlinear differential equations with $\Lambda(\xi)$ remaining as a parameter. This truncated approach of solving boundary-layer equations governed by partial differential equations is well established in the literature and is known as local similarity approach. The local similarity solutions give first-order estimate of the heat- and mass-transfer rates in boundary layers.[10,11]

In the local nonsimilarity approach, Eqs. (9-12) are converted to a system of coupled nonlinear ordinary differential equations and are solved in a manner similar to that for the local similarity solutions. One of the strengths of this method is that the solution of any streamwise location can be obtained independently of the solutions at upstream locations. Since the method is well documented elsewhere,[12,13] only pertinent equations and boundary conditions relevant to the problem will be outlined.

Defining the following functions,

$$G(\xi,\eta)=\frac{\partial f}{\partial \xi},\quad H(\xi,\eta)=\frac{\partial g}{\partial \xi},\quad M(\xi,\eta)=\frac{\partial \omega}{\partial \xi} \tag{21}$$

and substituting these new functions into Eqs. (9-12), leads to a set of ordinary differential equations. However, the introduction of the three new dependent variables in the problem requires three additional equations with appropriate boundary conditions. These equations were obtained by differentiating Eqs. (9-12), with respect to ξ. Using the foregoing results, the governing equations and the boundary conditions for the two-equation local nonsimilarity (LNS) model were obtained in the following form:

$$f'''+ff''+\Lambda(\xi)[-(f')^2+q+N\omega]=2\xi(f'G'-f''G) \tag{22}$$

$$q''+fq'Pr=2\xi(f'H-q'G)Pr \tag{23}$$

$$\omega''+f\omega'Sc=2\xi(f'M-\omega'G)Sc \tag{24}$$

$$G'''+fG''-2f'G'+3f''G+(\mathrm{d}\Lambda/\mathrm{d}\xi)[-(f')^2+q+N\omega]$$
$$+\Lambda(\xi)[-2f'G'+H+NM]-2\xi(G'G'-G''G)$$
$$=2\xi\left[f'\left(\frac{\partial G'}{\partial \xi}\right)-f''\left(\frac{\partial G}{\partial \xi}\right)\right] \tag{25}$$

$$H''/Pr-2f'H+3Gq'+fH'-2\xi(G'H-H'G)$$
$$=2\xi\left[f'\left(\frac{\partial H}{\partial \xi}\right)-q'\left(\frac{\partial G}{\partial \xi}\right)\right] \tag{26}$$

$$\omega''/Sc-2f'M+3G\omega'+fM'-2\xi(G'M-M'G)$$
$$=2\xi\left[f'\left(\frac{\partial M}{\partial \xi}\right)-\omega'\left(\frac{\partial G}{\partial \xi}\right)\right] \tag{27}$$

It is to be noted that the terms in the right-hand sides of Eqs. (25-27) have been neglected in parlance with the two-equation model local nonsimilarity approach as they are assumed to be very small.[13] This assumption is also valid for the present problem since the three-equation model (not shown) of the local nonsimilarity approach applied to a few specific cases of the problem only marginally improved the results over those from the two-equation model. Hence this approach was discarded in favor of the latter due to the excessive computing cost required to solve the three-equation set. The boundary conditions become

$$f(\xi,0)=f'(\xi,0)=G(\xi,0)=G'(\xi,0)=H(\xi,0)=M(\xi,0)=0$$

$$q(\xi,0)=\omega(\xi,0)=1$$

$$f'(\xi,\infty)=G'(\xi,\infty)=q(\xi,\infty)=H(\xi,\infty)$$
$$=\omega(\xi,\infty)=M(\xi,\infty)=0 \tag{28}$$

The coupled seventh-order system of ordinary nonlinear differential equations obtained from the local similarity (LS) approach and the fourteenth-order set obtained from the two-equation model nonsimilarity (LNS) approach after truncating the ξ-derivative terms from the equations were solved numerically by the Runge-Kutta Gill integration scheme along with the modified Newton-Raphson shooting method. Initial values of $f''(\xi,0)$, $q'(\xi,0)$, and $\omega'(\xi,0)$ for the LS set, Eqs. (18-20), and $f''(\xi,0)$, $q'(\xi,0)$, $\omega'(\xi,0)$, $G''(\xi,0)$, $H'(\xi,0)$, and $M'(\xi,0)$ for the LNS set, Eqs. (22-28), without the right-hand sides of Eqs. (25-27) are unknown. A forward integration procedure requires that all of the boundary conditions be specified at the starting point of integration. Therefore initial values of the above quantities were guessed along with the step size and a suitable upper bound of integration for the independent variable η. Subsequent initial values were obtained by modified Newton-Raphson method. Initial values at $\eta=0$ were refined until the conditions at the edge of the boundary layer (i.e., at $\eta=\eta_\infty$) $|f'(\xi,\eta)|$, $|f''(\xi,\eta)|$, $|q(\xi,\eta)|$, $|q'(\xi,\eta)|$, $|\omega'(\xi,\eta)|$, and $|\omega(\xi,\eta)|$ for the LS set and in addition to the above $|G''(\xi,\eta)|$, $|G'(\xi,\eta)|$, $|H(\xi,\eta)|$, $|H'(\xi,\eta)|$, $|M(\xi,\eta)|$, and $|M'(\xi,\eta)|$ for the LNS set were each of the order of $10^{-5}\sim10^{-6}$. In particular it was found advantageous to start the computations with a relatively small value of $\eta=\eta_\infty$ and then to increase its value successively until preassigned tolerances on the values of the variables at η_∞ are satisfied. For all the cases initial η_∞ were selected as 2.0 and were progressively increased with a step size of 1.0 or 2.0. A uniform integration step size of 0.04 was found to provide sufficiently accurate numerical results. The final upper limit of integration of $\eta_\infty=25$ satisfied the error condition fully for all cases studied.

The parameters of the problem are Prandtl number, Pr, Schmidt number, Sc, and the ratio of the species to the thermal buoyancy force, N. Computations were carried out for the diffusion of water vapor and naphthalene into air. Both of these diffusing species have practical as well as technological applications. Calculations were carried out when the thermal buoyancy force was both opposing and aiding the concentration buoyancy force. The case $N=0$, corresponds to the pure thermal convection; when $N<0$, the thermal and concentration buoyancy forces oppose each other, while for $N>0$, they assist each other in driving the flow. The values of R may assume any positive or negative value since the quantities, β, β^*, (T_w-T_∞), and (C_w-C_∞) may be positive or negative.

The physical quantities of interest include the local Sherwood number, Sh_D, the local Nusselt number, Nu_D, and the local wall shear stress, τ_w, defined, respectively, by

$$Sh_D=\dot{m}_wD/[\rho D^*(C_w-C_\infty)] \tag{29a}$$

$$Nu_D=q_wD/[k(T_w-T_\infty)] \tag{29b}$$

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{29c}$$

From Fick's law, $\dot{m}_w=-\rho D^*(\partial C/\partial y)_{y=0}$, and from Fourier's law, $q=-k(\partial T/\partial y)_{y=0}$, along with Eqs. (6-9), it

Table 1 Local heat-transfer parameter, $Nu_D/Gr_D^{1/4}$, for isothermal, horizontal circular cylinder evaluated by various predictive methods ($Pr = 0.70$)

	Various series solutions			Finite difference	Present analyses	
θ, deg	Blasius[14]	Merk[15]	Görtler[15]	Merkin[16]	LS	LNS
0	0.4402	0.4402	0.4402	0.4402	0.44021	0.44021
30	0.4350	0.4346	0.4351	0.4348	0.43476	0.43502
60	0.4192	0.4186	0.4187	0.4190	0.41913	0.41926
90	0.3930	0.3913	0.3898	0.3922	0.39252	0.39217
120	0.3563	0.3508	0.3449	0.3522	0.35271	0.35407
130	0.3417	0.3338	0.3251	0.3370	0.33583	0.33963
150	0.3091	0.2933	0.2740	0.2986	0.29241	0.31661
160	0.2928	0.2700	—	0.2752	0.26612	0.32195

Table 2 The local mass transfer parameter, $Sh_D/(Gr_{D,c})^{1/4}$, the local heat transfer parameter, $Nu_D/Gr_D^{1/4}$, and the local wall shear stress parameter, $\tau_w D^2/[\rho\nu^2(Gr_D)^{3/4}]$ ($Pr = 0.72$)

			$Sh_D/(Gr_{D,c})^{1/4}$		$Nu_D/(Gr_D)^{1/4}$		$\tau_w D^2/[\rho\nu^2(Gr_D)^{3/4}]$	
Sc	N	θ, deg	LS	LNS	LS	LNS	LS	LNS
0.63	−0.5	0	0.40623	0.40623	0.36638	0.36638	0	0
		30	0.40148	0.40131	0.36209	0.36196	0.21316	0.21342
		60	0.38710	0.38659	0.35829	0.35865	0.38885	0.38656
		90	0.36240	0.36147	0.32648	0.32600	0.48528	0.49415
		120	0.32559	0.32560	0.29364	0.29383	0.48210	0.50866
		130	0.30983	0.31113	0.27937	0.28126	0.45461	0.49229
		150	0.28931	0.28492	0.24327	0.26031	0.35404	0.43338
		160	0.24242	—	0.21853	—	0.27671	—
	0.5	0	0.55034	0.55034	0.49542	0.49542	0	0
		30	0.52992	0.54378	0.48964	0.48952	0.49816	0.49879
		60	0.52447	0.52394	0.47212	0.47163	0.90379	0.90922
		90	0.49120	0.49029	0.44214	0.44133	1.13553	1.15663
		120	0.44162	0.44244	0.39749	0.39862	1.12984	1.19340
		130	0.42035	0.42401	0.37831	0.38255	1.06619	1.15698
		150	0.36637	0.39207	0.32968	0.35791	0.83203	1.02709
		160	0.32928	—	0.29629	—	0.65124	—
	2.0	0	0.46571	0.46571	0.59269	0.59269	0	0
		30	0.46029	0.46017	0.58578	0.58562	0.84287	0.84394
		60	0.44384	0.44340	0.56483	0.56426	1.52934	1.53856
		90	0.41571	0.41499	0.52899	0.52809	1.92186	1.95769
		120	0.37380	0.36490	0.47561	0.47724	1.91287	2.02117
		130	0.35581	0.35926	0.45271	0.45821	1.80548	1.96038
		150	0.31017	0.33312	0.39458	0.42984	1.40958	1.74432
		160	0.27881	—	0.35466	—	1.10368	—
2.57	−0.5	0	0.85918	0.85918	0.41256	0.41256	0	0
		30	0.84910	0.84885	0.40778	0.40769	0.25285	0.25319
		60	0.81858	0.81779	0.39326	0.39297	0.45932	0.46230
		90	0.76636	0.76663	0.36845	0.36820	0.57849	0.59049
		120	0.68862	0.70538	0.33147	0.33431	0.57800	0.61741
		130	0.65529	0.69354	0.31560	0.32258	0.54653	0.60602
		150	0.57079	—	0.27529	—	0.42881	—
		160	0.51284	—	0.24757	—	0.33689	—
	0.5	0	1.00711	1.00711	0.47014	0.47014	0	0
		30	0.99513	0.99472	0.46463	0.46449	0.46678	0.46735
		60	0.95883	0.95738	0.41933	0.44737	0.84624	0.85114
		90	0.89673	0.89516	0.41933	0.48123	1.06176	1.06708
		120	0.80432	0.81575	0.37675	0.37630	1.05386	1.10736
		130	0.75736	0.79460	0.35848	0.35966	0.99334	1.06708
		150	0.66456	—	0.31216	—	0.77269	—
		160	0.59614	—	0.28039	—	0.60345	—
	2.0	0	0.81194	0.81194	0.52591	0.52591	0	0
		30	0.80148	0.80112	0.51972	0.51954	0.73567	0.73654
		60	0.77204	0.77070	0.50093	0.50020	1.33273	1.34014
		90	0.72169	0.71971	0.46879	0.46710	1.66973	1.69734
		120	0.64676	0.63321	0.42090	0.41822	1.65311	1.72776
		130	0.61467	0.63248	0.40039	0.39757	1.55627	1.65456
		150	0.53356	—	0.34837	—	1.20651	—
		160	0.47825	—	0.31274	—	0.94004	—

can be shown that

$$Sh_D(Gr_{D,c})^{-1/4} = -\sin X\omega'(\xi,0)/[|R|^{1/4}\sqrt{\xi}] \tag{30}$$

$$Nu_D(Gr_D)^{-1/4} = -\sin Xq'(\xi,0)/\sqrt{\xi} \tag{31}$$

$$\tau_w D^2/[\rho v^2(Gr_D)^{3/4}] = \sqrt{2/\xi}\sin^2(X)f''(\xi,0) \tag{32}$$

The general nondimensional form of boundary-layer thickness is defined by

$$\delta_x = y_x Gr_D^{1/4}/D = \eta_x[2\xi/(1-\cos\theta)]^{1/2} \tag{33}$$

where x corresponds to c, t, or v and η_x corresponds to the distances inside the boundary layers such that

$$\omega(\xi,\eta_c) = q(\xi,\eta_t) = f'(\xi,\eta_v) = 0.01 \tag{34}$$

Results and Discussion

Pure free-convective heat-transfer analyses around a horizontal cylinder have been carried out by the local similarity (LS) as well as by the local nonsimilarity (LNS) approaches. The heat-transfer results obtained from the present investigations along with the results of other investigators are reported in Table 2 in the form of $Nu_D/(Gr_D)^{1/4}$. The above solutions were carried out to check the present numerical solutions against the available literature results and also to provide a basis for comparison with the combined heat- and mass-transfer solutions.

A comparison of the results of $Nu_D/(Gr_D)^{1/4}$ in Table 1 shows that up to $\theta = 130$ deg the present LS and LNS results agree quite closely with those of the series as well as finite difference (FD) solutions reported in prior studies.

Beyond $\theta = 130$ deg LS solutions produce reasonably accurate results compared to the series and FD solutions. For example, at $\theta = 160$ deg, the differences in the present LS result when compared with the Merk-type series solution[15] and FD[16] solution are only 1.4 and 3.3%, respectively. On the contrary, LNS yields results that are inaccurate and inconsistent beyond $\theta = 130$ deg. The latter observation is noteworthy since it has been shown by several authors in a number of studies of free-convective problems[9,11] that the LNS scheme yields results that are more accurate than the LS results. The reason for this unexpected result in the present LNS solutions at higher values of θ could not be traced. One of the reviewers pointed out the existence of a numerical study[17] on pure free-convective heat transfer around a horizontal cylinder where similar levels of inaccuracy in the LNS solutions were observed beyond $\theta = 130$ deg. No reason was cited for this unexpected trend observed in the LNS solutions.

In light of the above observations, numerical solutions of combined heat and mass transfer around a horizontal cylinder were carried out up to $\theta = 150$ deg from the front stagnation point by the LNS scheme and up to $\theta = 160$ deg by the LS scheme. Due to space limitations, and in order to permit future comparative evaluation of these results, all results are reported in a tabular form in Table 2. In Table 3 the corresponding dimensionless boundary-layer thicknesses from LS solutions are tabulated for ease of interpretation of the results of Table 2.

The results for the local Sherwood number in the form $Sh_D/(Gr_{D,c})^{1/4}$ are shown in columns 4 and 5 of Table 2. These results reveal that for a given buoyancy force maximum mass transfer occurs at the lower stagnation point and that rate decreases progressively with angular distance from the lower stagnation point. Note that in the fore portion of the cylinder the rate of decrease of mass transfer is very sluggish while in the aft portion the rate of decrease is rapid. From entries for the dimensionless concentration boundary-layer thicknesses, δ_c in Table 3, it is seen that the boundary-layer thickness increases very slowly in the first half-portion of the cylinder. In the second half-portion the increase of the boundary-layer thickness is rapid and seems to separate near the rear pole. The present method of analysis breaks down toward the rear end as the boundary layer thickens rapidly in this region and is infinite at $\theta = 180$ deg. The thicker the δ_c, the less is the concentration gradient at the surface of the cylinder; this in turn causes the mass-transfer rate to decrease.

Table 2 further reveals that large $Sh_D/(Gr_{D,c})^{1/4}$ is associated with larger Sc. A larger Sc corresponds to a smaller binary diffusion coefficient in a given binary mixture and hence a thinner δ_c relative to the momentum boundary-layer thickness, δ_c. This results in a larger concentration gradient at the wall, which in turn enhances the mass-transfer rate.

Table 3 Dimensionless concentration, thermal and momentum boundary-layer thicknesses; local similarity solution, $Pr = 0.72$

Sc	R	θ deg	δ_c	δ_t	δ_v
0.63	−0.5	0	6.926	6.251	6.011
		30	7.006	6.323	6.079
		60	7.244	6.553	6.247
		90	7.760	6.999	6.674
		120	8.637	7.882	7.269
		130	9.070	8.174	7.572
		150	10.399	9.375	8.445
		160	11.563	10.425	9.149
	0.5	0	4.949	4.485	5.501
		30	5.006	4.537	5.555
		60	5.193	4.702	5.735
		90	5.542	5.019	6.074
		120	6.163	5.580	6.622
		130	6.471	5.859	6.790
		150	7.419	6.717	7.642
		160	8.253	7.470	8.250
	2.0	0	4.111	3.727	4.941
		30	4.159	3.771	4.997
		60	4.313	3.908	5.143
		90	4.604	4.172	5.446
		120	5.117	4.638	5.945
		130	5.378	4.872	6.193
		150	6.161	5.583	6.867
		160	6.853	6.209	7.430
2.57	−0.5	0	2.497	5.267	6.116
		30	2.526	5.328	6.186
		60	2.618	5.526	6.186
		90	2.795	5.896	6.741
		120	3.103	6.585	7.349
		130	3.259	6.877	7.642
		150	3.733	7.883	8.478
		160	4.149	8.762	9.164
	0.5	0	2.207	4.846	5.574
		30	2.230	4.901	5.629
		60	2.313	5.083	5.808
		90	2.468	5.427	6.141
		120	2.745	6.032	6.698
		130	2.864	6.350	7.116
		150	3.309	7.266	7.730
		160	3.682	8.083	8.350
	2.0	0	1.976	4.482	5.103
		30	1.999	4.535	5.154
		60	2.073	4.701	5.310
		90	2.214	5.019	5.618
		120	2.464	5.582	6.127
		130	2.587	5.863	6.369
		150	2.971	6.724	7.071
		160	3.306	7.480	7.635

Dependence of $Sh_D/(Gr_{D,c})^{1/4}$ on N is quite interesting. From Table 2 it is seen that in the case of diffusion of naphthalene ($Sc=2.57$) into air with $N=-0.5$ or 0.5, local mass-transfer rates all around the cylinder are higher than the corresponding mass-transfer rates for $N=2.0$. From Table 3 it is seen that the δ_c is thinner for $N=2.0$ than for $N=0.5$ or -0.5. So it is expected that the concentration gradient will be higher at the surface for the former case than for the latter cases. This proves to be true. From the numerical calculations it is seen that at a polar angle of $\theta=120$ deg for diffusion of naphthalene into air, the concentration gradients at the surface of the cylinder with $N=2$, 0.5 and -0.5 using the LS scheme are -0.74681, -0.65672, and -0.56226, respectively. But $Sh_D/(Gr_{D,c})^{1/4}$, apart from the concentration gradient at the wall $(-\omega'(\xi,))$, also depends on $|N|^{1/4}$. (Note that the absolute value of $|Gr_{D,c}/N|$ is needed to ensure that Sh_D is positive since $Gr_{D,c}$ and N can assume positive as well as negative values.) For a very small value of N, either positive or negative, the factor $|1/N|^{1/4}$ takes on a very high value and hence results in higher $Sh_D/(Gr_{D,c})^{1/4}$. Depending on Sc, $Sh_D/(Gr_{D,c})^{1/4}$ at small N may be higher than that at high values of N. This may appear somewhat surprising, but physical interpretation can elucidate the computed results. Physically, for a low value of N, the flow condition is induced almost entirely by the thermal buoyancy force, and the species diffusion mechanism becomes very effective at very low concentration levels. For a fixed $Gr_{D,c}$ a small value of N implies a large value of Gr_D. Thus, large $Sh_D/(Gr_{D,c})^{1/4}$ results when Gr_D is large compared to $Gr_{D,c}$.

The local Nusselt number results in the form $Nu_D/(Gr_D)^{1/4}$ are displayed in columns 6 and 7 of Table 2. The results show a similar behavior as that of the $Sh_D/(Gr_{D,c})^{1/4}$ results, i.e., the maximum heat transfer occurs at the lower pole and $Nu_D/(Gr_D)^{1/4}$ decreases slowly around the surface of the cylinder up to the great circle, after which it decreases increasingly rapidly as one approaches the top stagnation point. Examination of the thermal boundary-layer thickness, δ_t in Table 3 shows that the thermal boundary layer thickens with the polar angle θ and its rate increases rapidly beyond $\theta=90$ deg. As with the effect of δ_c on the mass-transfer rates, with the increase of δ_c the temperature gradient at the surface of the cylinder decreases, which results in lower heat-transfer rates. Note that in order to conserve space pure heat-transfer results (i.e., for $N=0$) have not been reported in Table 2 for $Pr=0.72$. The pure heat-transfer results reported in Table 1 for $Pr=0.70$ varied at most 1.5% from the corresponding results for $Pr=0.72$. Therefore the results of Table 1 could be taken as the representative results for the latter case for the sake of comparison. Table 2 further reveals that $Nu_D/(Gr_D)^{1/4}$ increases for aiding buoyancy forces, i.e., for $N>0$ and decreases for opposing buoyancy forces, i.e., for $N<0$. Observe that $Nu_D/(Gr_D)^{1/4}$ increases and decreases beyond the values for $N=0$ when the buoyancy force form species diffusion assists and opposes, respectively, the thermal buoyancy force. Table 2 further indicates that larger departures of $Nu_D/(Gr_D)^{1/4}$ from $N=0$ are associated with smaller values of Sc for both $N>0$ and $N<0$. This trend can be physically interpreted in terms of the fact that diffusing species with smaller Sc possess a larger binary diffusion coefficient which exerts a larger effect on the thermal fields.

Table 2 displays the fact that for a fixed Pr, Nu_D decreases with increasing Sc for aiding flows, i.e., $N>0$, while Nu_D increases with increasing Sc for counterbuoyant flows, i.e., $N<0$. The reason for this reversed effect of Sc on the thermal diffusion mechanism can be explained from the observed fact by comparing the corresponding changes in the thickness of the boundary layers in Table 3. For $N>0$ with the increase of Sc, δ_c decreases due to the lower diffusion coefficient associated with the higher Sc. Since, in the coupled convective flow, concentration and thermal fields are intimately related to the flowfield, the adverse effect of one tends to be balanced by the other two. This reduction in δ_c causes the momentum boundary-layer thickness δ_v to adjust itself by expanding its depth. Since in the problem the thermal field is intimately coupled with the flowfield, δ_v also forces δ_t to follow its expansion. This results in a greater δ_t, thereby reducing the thermal gradients at the wall and also the Nu_D. For example, for $Sc=0.63$ with $N=0.5$ and at $\theta=90$ deg, $Nu_D/(Gr_D)^{1/4}$ is found to be 0.39747 and the δ_c, δ_t, and δ_v are 6.17, 5.58, and 6.52, respectively. With $Sc=2.57$, with the same N and at the same θ, $Nu_D/(Gr_D)^{1/4}$ assumes a value of 0.37675 and the corresponding δs are 2.74, 6.04, and 6.70, respectively, thereby conforming to the physical reality. The reverse phenomenon that prevails for $N<0$ can be similarly explained.

The results of the local wall shear stress reported in the form $\tau_w D^2/[\rho\nu^2(Gr_D)^{3/4}]$ in columns 8 and 9 of Table 2 show trends similar to those for the heat-transfer results, i.e., the wall shear stress increases beyond the values for $N=0$ when the buoyancy force from mass diffusion acts in the same direction as the thermal buoyancy force to assist the flow and decreases beyond $N=0$ when the two buoyancy forces are counterbuoyant. From Tables 2 and 3 it is seen that smaller Sc (associated with larger binary diffusion coefficient) exerts a greater influence on the flowfield; as a result a larger departure of wall shear stress from $N=0$ occurs for both $N>0$ and $N<0$.

Conclusions

Under the present formulation of the problem of combined free-convective mass and heat transfer around a horizontal cylinder, it is observed that a simple local similarity approach provides reasonably accurate solutions compared to the other conventional approaches, thus suggesting its utility for practical applications when approximate results suffice. Computationally much more demanding solutions from the LNS approach generate results, especially at large angular distances from the stagnation point, that are inaccurate and inconsistent. Analyses show that, as with pure thermal and pure species transfer in free convection, the maximum local mass- and heat-transfer rates in the case of simultaneous transfer around the cylinder occur at the lower stagnation point. The heat- and mass-transfer rates decrease progressively with increase of the angular position from the stagnation point. The rates decrease slowly on the lower part of the cylinder, but over the upper part they decrease at a much faster rate with angular position.

Both Nu_D and the τ_w increase and decrease from their respective pure thermal free-convection values as the buoyancy forces from species diffusion assist and oppose, respectively, the thermal buoyancy force. The departures of these two quantities from the pure thermal convection results are more pronounced at low Schmidt numbers. For a fixed $Gr_{D,c}$ the mass-transfer rate is found to increase as the thermal buoyancy force increases. While the local surface heat transfer and local wall shear stress are enhanced for diffusion of lower molecular weight gases and vapors, the surface mass-transfer rate increases for high molecular weight gases and vapors.

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